G3 MATHEMATICS HOLIDAY REVISION

Week 0

Foundation diagnostic

Worked solutions

Upper Secondary - Sec 3 and Sec 4

18 original questions

Attempt the student worksheet first. Then compare each step with the reasoning and checks in this companion.

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WORKED SOLUTIONS

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READINESS CHECKS

Q1

[2 marks]

Simplify $(3x^4y^{-2})(2x^{-1}y^5)$ using positive indices. State restrictions.

WORKED REASONING

1. Use the product law separately for x and y.

2. Coefficient: $3\times2=6$.

3. x exponent: $4+(-1)=3$; y exponent: $-2+5=3$.

4. Answer: $6x^3y^3$. Negative exponents in the original require $x\neq0$ and $y\neq0$.

WORKED SOLUTIONS

Week 0 · Foundation diagnostic

READINESS CHECKS

Q2

[2 marks]

Rewrite $8^{2/3}$ as an integer. Explain the denominator of the index.

WORKED REASONING

1. 2/3 means cube root, then square (or square then cube root).

2. $\sqrt[3]{8}=2$, so $8^{2/3}=2^2=4$.

WORKED SOLUTIONS

Week 0 · Foundation diagnostic

READINESS CHECKS

Q3

[2 marks]

Factorise completely $12u^2 - 27v^2$.

WORKED REASONING

1. Take out 3: $3(4u^2-9v^2)$.

2. Difference of squares: $4u^2-9v^2=(2u-3v)(2u+3v)$.

3. Answer: $3(2u-3v)(2u+3v)$.

WORKED SOLUTIONS

Week 0 · Foundation diagnostic

CORE REPAIR

Q4

[4 marks]

Simplify $(15a^3b^{-2})/(8c) \div (25a^{-1}b)/(12c^2)$ using positive indices. State restrictions.

WORKED REASONING

1. Change division to multiplication by the reciprocal of the second fraction.

2. $(15a^3b^{-2})/(8c) \times (12c^2)/(25a^{-1}b)$.

3. Coefficients: $180/200=9/10$.

4. a exponent $3-(-1)=4$; b exponent $-2-1=-3$; c exponent $-1+2=1$.

5. Answer: $9a^4c/(10b^3)$. Restrictions: $a,b,c\neq0$.

WORKED SOLUTIONS

Week 0 · Foundation diagnostic

CORE REPAIR

Q5

[3 marks]

Given $2^{x+1} \times 4^{x-2} = 32$, solve for x. Show common bases and check.

WORKED REASONING

1. Rewrite all terms with base 2: $4=2^2$ and $32=2^5$.

2. $2^{x+1} \times 2^{2x-4} = 2^5$, hence $3x-3=5$.

3. Add 3: $3x=8$, so $x=8/3$.

4. Check: at $x=8/3$, $(x+1)+2(x-2)=11/3+4/3=5$, so the left side is $2^5=32$.

WORKED SOLUTIONS

Week 0 · Foundation diagnostic

CORE REPAIR

Q6

[3 marks]

Rectangle sides are $3x^2y$ and $2xy^{-1}$, with $x>0, y>0$. Find area and perimeter.

Explain why perimeter terms cannot combine.

WORKED REASONING

1. Area = $(3x^2y)(2xy^{-1})=6x^3$.

2. Perimeter $=2(3x^2y+2xy^{-1})=6x^2y+4xy^{-1}$.

3. $x,y>0$ ensures both side lengths are positive. The terms have different powers of x and y, so they are unlike terms and cannot combine.

WORKED SOLUTIONS

Week 0 · Foundation diagnostic

CORE REPAIR

Q7

[3 marks]

For positive m,n, factorise $18m^3n - 8mn^3$ completely. Hence solve $18m^3n - 8mn^3 = 0$ for n in terms of m.

WORKED REASONING

1. Factor 2mn: $2mn(9m^2-4n^2)$.

2. Apply difference of squares: $2mn(3m-2n)(3m+2n)$.

3. For positive m,n, 2mn and 3m+2n are nonzero. Thus $3m-2n=0$, so $n=3m/2$.

WORKED SOLUTIONS

Week 0 · Foundation diagnostic

CORE REPAIR

Q8

[3 marks]

Simplify 3/(2x−1) − 7/(x+1) as one fraction. State excluded values.

WORKED REASONING

1. Common denominator: (2x−1)(x+1).

2. Numerator: 3(x+1)−7(2x−1)=3x+3−14x+7=10−11x.

3. Answer: (10−11x)/((2x−1)(x+1)).

4. Original denominators require x≠1/2 and x≠−1.

WORKED SOLUTIONS

Week 0 · Foundation diagnostic

MULTISTEP PRACTICE

Q9

[4 marks]

Simplify $[(12x^2y^{-1})/(5z)] \div [(18x^{-1}y^2)/(25z^2)]$ using positive indices. State restrictions, then evaluate at $x=-1, y=2, z=3$.

WORKED REASONING

1. Invert only the divisor: first fraction × $(25z^2)/(18x^{-1}y^2)$.

2. Coefficient: $(12\times25)/(5\times18)=10/3$.

3. x exponent $2-(-1)=3$; y exponent $-1-2=-3$; z exponent $-1+2=1$.

4. Answer: $10x^3z/(3y^3)$, with $x,y,z\neq0$.

5. At $(-1,2,3)$: $10(-1)^3(3)/(3\cdot2^3)=-30/24=-5/4$.

WORKED SOLUTIONS

Week 0 · Foundation diagnostic

MULTISTEP PRACTICE

Q10

[4 marks]

Solve $3^{2x-1} = 27^{x-2} \times 9^2$. Explain why common-base comparison is reversible.

WORKED REASONING

1. Use $27=3^3$ and $9^2=3^4$. RHS becomes $3^{3(x-2)+4}=3^{3x-2}$.
2. LHS is 3^{2x-1}. Since base 3 is positive and not 1, equal powers imply equal exponents.
3. $2x-1=3x-2$, so $x=1$.
4. Check: LHS=3; RHS=$27^{-1}\times 81=3$.

WORKED SOLUTIONS

Week 0 · Foundation diagnostic

MULTISTEP PRACTICE

Q11

[4 marks]

$A=2^r3^s$ and $B=2^t3^u$, with r,s,t,u non-negative integers and A/B an integer. Give necessary and sufficient conditions for A/B to be (i) a positive integer square, (ii) a positive integer cube. Explain.

WORKED REASONING

1. $A/B=2^{r-t}3^{s-u}$. Since A/B is an integer, both differences are non-negative integers.
2. It is a positive integer square iff both differences are even.
3. It is a positive integer cube iff both differences are multiples of 3.
4. Non-negativity is needed for integer quotient; parity/multiples ensure square/cube exponents.

WORKED SOLUTIONS

Week 0 · Foundation diagnostic

MULTISTEP PRACTICE

Q12

[4 marks]

$A=(x+2)^2-(x-2)^2$. Simplify by expansion and by an identity; explain cancellation, then find x when $A=40$.

WORKED REASONING

1. Expansion: $(x^2+4x+4)-(x^2-4x+4)=8x$.

2. Difference of squares: $[(x+2)-(x-2)][(x+2)+(x-2)]=(4)(2x)=8x$.

3. The x^2 and constant terms cancel; the cross terms remain.

4. Set $8x=40$, giving $x=5$.

WORKED SOLUTIONS

Week 0 · Foundation diagnostic

MULTISTEP PRACTICE

Q13

[4 marks]

N=$2^3 \times 3^2 \times 5$. Find least positive k,j so Nk is a perfect square and Nj a perfect cube. Explain exponent adjustments.

WORKED REASONING

1. For a square, every prime exponent must be even. 2^3 needs one extra 2; 3^2 is already even; 5^1 needs one extra 5.
2. Least k=2×5=10, giving exponents 4,2,2.
3. For a cube, exponents must be multiples of 3: 2^3 is already suitable; add one 3 and 5^2, so j=3×25=75.
4. Answer: k=10 and j=75.

WORKED SOLUTIONS

Week 0 · Foundation diagnostic

SPOT THE MISTAKE

Q14

[3 marks]

A student writes $x^{-2}-x^{-3}=x$. Find the correct simplification and domain.

Identify the mistaken exponent operation.

WORKED REASONING

1. Write with common denominator x^3: $1/x^2-1/x^3=(x-1)/x^3$.

2. Domain: $x\neq0$, from negative powers.

3. The exponent subtraction law applies to division of like bases, not subtraction.

WORKED SOLUTIONS

Week 0 · Foundation diagnostic

SPOT THE MISTAKE

Q15

[3 marks]

A student expands $(2r-3)^2$ as $4r^2+9$. Correct it and explain the missing term.

WORKED REASONING

1. $(2r-3)^2=(2r)^2-2(2r)(3)+3^2$.

2. So the result is $4r^2-12r+9$. The middle term is twice the product of the two terms, with the sign from -3.

WORKED SOLUTIONS

Week 0 · Foundation diagnostic

SPOT THE MISTAKE

Q16

[3 marks]

A student simplifies x(x+4)/(x+4) to x+4. Give correct simplification and restriction. Explain what factor is cancelled.

WORKED REASONING

1. Factor numerator and denominator as x·(x+4)/(x+4).

2. For x≠−4, cancel the common factor x+4.

3. Answer: x, with original restriction x≠−4. Cancellation does not leave x+4 behind.

WORKED SOLUTIONS

Week 0 · Foundation diagnostic

STRETCH

Q17

[5 marks]

Find the least positive n such that 72n is both a perfect square and a perfect cube. Justify the prime-exponent conditions.

WORKED REASONING

1. $72=2^3\times3^2$. For a number to be both a square and cube, each prime exponent must be a multiple of lcm(2,3)=6.
2. Raise the 2 exponent from 3 to 6: multiply by 2^3. Raise 3 from 2 to 6: multiply by 3^4.
3. Least $n=2^3\times3^4=8\times81=648$. Then $72n=2^6\times3^6=6^6$, which is both a square and cube.

WORKED SOLUTIONS

Week 0 · Foundation diagnostic

STRETCH

Q18

[4 marks]

Solve $2^{x+1}=8^{2-x}$. Explain why common-base comparison works and whether any real x is excluded.

WORKED REASONING

1. Rewrite 8 as 2^3: RHS=$2^{3(2-x)}=2^{6-3x}$.

2. Equal powers of base 2 give $x+1=6-3x$; hence $4x=5$, $x=5/4$.

3. Both sides are defined for every real x; no values are excluded.