G3 MATHEMATICS HOLIDAY REVISION

Week 1

Intensive algebra

Worked solutions

Upper Secondary - Sec 3 and Sec 4

19 original questions

Attempt the student worksheet first. Then compare each step with the reasoning and checks in this companion.

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WORKED SOLUTIONS

Week 1 · Intensive algebra

WARM-UPS

Q1

[2 marks]

Factorise $x^2-5x-14$ completely. State its zeros.

WORKED REASONING

Two numbers with product -14 and sum -5 are -7 and 2, so $x^2-5x-14=(x-7)(x+2)$.

Zeros: $x=7$ or $x=-2$.

WORKED SOLUTIONS

Week 1 · Intensive algebra

WARM-UPS

Q2

[2 marks]

Evaluate $16^{3/4}$ without a calculator. Explain which operation the denominator 4 represents.

WORKED REASONING

The denominator means fourth root: $\sqrt[4]{16}=2$; cube that result to obtain $2^3=8$.

WORKED SOLUTIONS

Week 1 · Intensive algebra

WARM-UPS

Q3

[2 marks]

Simplify $(x^2-9)/(x-3)$ and give its original restriction. Can the simplified expression give a value for the original fraction at $x=3$?

WORKED REASONING

$(x-3)(x+3)/(x-3)=x+3$ for $x\neq 3$. The original fraction is undefined at 3 even though $x+3$ has a value there, so substitution at 3 is invalid for the original expression.

WORKED SOLUTIONS

Week 1 · Intensive algebra

CORE APPLICATIONS

Q4

[4 marks]

Factorise $6x^2-x-2$. Hence solve $6x^2-x-2=0$ and check one root by substitution.

WORKED REASONING

Split the middle term using product -12 and sum -1:

$6x^2+3x-4x-2=3x(2x+1)-2(2x+1)=(3x-2)(2x+1)$. Hence $x=2/3$ or $x=-1/2$. Check $x=2/3$:

$6(4/9)-2/3-2=8/3-2/3-2=0$.

WORKED SOLUTIONS

Week 1 · Intensive algebra

CORE APPLICATIONS

Q5

[4 marks]

Write $2/(x-1)-3/(x+2)$ as one simplified fraction. State every excluded value.

WORKED REASONING

Use $(x-1)(x+2)$ as the common denominator. Numerator $2(x+2)-3(x-1)=2x+4-3x+3=7-x$.

Thus $(7-x)/((x-1)(x+2))$, with $x \neq 1,-2$. The minus sign applies to the whole second numerator.

WORKED SOLUTIONS

Week 1 · Intensive algebra

CORE APPLICATIONS

Q6

[4 marks]

Simplify $[(x^2-9)/(x^2+5x+6)] \div [(x-3)/(x+2)]$. Give all restrictions from the original expression, including the condition that the divisor is nonzero.

WORKED REASONING

Factor: first fraction is $(x-3)(x+3)/[(x+2)(x+3)]$. Multiply by reciprocal $(x+2)/(x-3)$ to obtain 1. Original denominators exclude $x=-2,-3$; the divisor is zero at $x=3$, so $x\neq 3$ as well. Answer 1 for $x\neq -3,-2,3$.

WORKED SOLUTIONS

Week 1 · Intensive algebra

CORE APPLICATIONS

Q7

[4 marks]

Solve $4^{x-1}=8^{2-x}$. Explain why comparing exponents is valid, then check your answer.

WORKED REASONING

Rewrite as $2^{2x-2}=2^{6-3x}$. Base 2 is positive and not 1, so equal powers have equal exponents: $2x-2=6-3x$, giving $x=8/5$. Check exponents: both equal 6/5, so both original sides equal $2^{6/5}$.

WORKED SOLUTIONS

Week 1 · Intensive algebra

CORE APPLICATIONS

Q8

[4 marks]

The quadratic $x^2+(k-3)x-3k$ has a root $x=-2$. Find k, factorise the quadratic and state its other root.

WORKED REASONING

Substituting -2 gives $4-2(k-3)-3k=10-5k=0$, so $k=2$. The quadratic becomes $x^2-x-6=(x-3)(x+2)$; its other root is $x=3$. Checking the original symbolic expression, $(x-3)(x+k)$ expands to $x^2+(k-3)x-3k$.

WORKED SOLUTIONS

Week 1 · Intensive algebra

CHALLENGING REASONING

Q9

[5 marks]

Solve $2/(x-1)+3/(x+2)=1$. State restrictions before clearing denominators and check that each candidate is allowed.

WORKED REASONING

Require $x \neq 1, -2$. Multiply by $(x-1)(x+2)$: $2(x+2)+3(x-1)=(x-1)(x+2)$. This gives $5x+1=x^2+x-2$, or $x^2-4x-3=0$. By the quadratic formula, $x=(4\pm\sqrt{28})/2=2\pm\sqrt{7}$. Neither is 1 or -2, so both are solutions. Each satisfies the cleared equation, and denominator multiplication was reversible on the permitted domain.

WORKED SOLUTIONS

Week 1 · Intensive algebra

CHALLENGING REASONING

Q10

[5 marks]

A rectangle has width $(x-1)$ cm and length $(x+4)$ cm, where $x>1$. Its area is 36 cm². Find x, both dimensions and the perimeter. Explain why the other algebraic root is rejected.

WORKED REASONING

$(x-1)(x+4)=36$ gives $x^2+3x-4=36$, hence $x^2+3x-40=(x+8)(x-5)=0$. Thus $x=5$ or -8. Since $x>1$, reject -8: it would give negative side lengths. Width 4 cm, length 9 cm, perimeter $2(4+9)=26$ cm. Check area $4\times9=36$ cm².

WORKED SOLUTIONS

Week 1 · Intensive algebra

CHALLENGING REASONING

Q11

[5 marks]

Find the least positive integer n for which both 12n is a perfect square and 18n is a perfect cube. Prove your choice is least.

WORKED REASONING

Write $n=2^a3^b$ for the least candidate; introducing other prime factors can only increase n and their exponents would also have to satisfy both power conditions. Since $12=2^2 3$, square condition requires a+2 even and b+1 even: a even, b odd. Since $18=2\cdot 3^2$, cube condition requires a+1 and b+2 divisible by 3: a leaves remainder 2 when divided by 3, and b leaves remainder 1 when divided by 3. Least nonnegative choices are a=2, b=1, so $n=2^2\cdot 3=12$. Check $12n=144=12^2$; $18n=216=6^3$. Any larger admissible exponents or extra primes make n larger.

WORKED SOLUTIONS

Week 1 · Intensive algebra

CHALLENGING REASONING

Q12

[5 marks]

Let $G=(x^2-4)/(x^2-x-6)+(x-2)/(x+2)$. Simplify G to one fraction, state every original restriction, then find all x for which $G=0$.

WORKED REASONING

Factor the first numerator and denominator: $(x-2)(x+2)/[(x-3)(x+2)]$. Original denominators require $x \neq -2,3$. For allowed x, the first term is $(x-2)/(x-3)$. Use denominator $(x-3)(x+2)$ for the sum: numerator $(x-2)(x+2)+(x-2)(x-3)=(x-2)(2x-1)$. Thus $G=(x-2)(2x-1)/[(x-3)(x+2)]$, with $x \neq -2,3$. A fraction is zero when its numerator is zero and denominator nonzero, so $x=2$ or $x=1/2$; both are permitted.

WORKED SOLUTIONS

Week 1 · Intensive algebra

CHALLENGING REASONING

Q13

[5 marks]

The equations $x^2-5x+6=0$ and $x^2-(k+1)x+2k=0$ have exactly one common real root. Find k and all roots of the second equation. Explain why the other root of the first equation cannot be common.

WORKED REASONING

Factor the first equation as $(x-2)(x-3)=0$, giving candidates 2 and 3. Substituting $x=2$ into the second left side gives $4-2(k+1)+2k=2$, so 2 is never a root. Substituting $x=3$ gives $9-3(k+1)+2k=6-k$; hence $k=6$. The second equation is then $x^2-7x+12=(x-3)(x-4)=0$, with roots 3 and 4. Its only shared root with {2,3} is 3.

WORKED SOLUTIONS

Week 1 · Intensive algebra

CHALLENGING REASONING

Q14

[5 marks]

Solve the simultaneous equations $2x+3y=17$ and $x/2-y/3=1$. Show why your chosen elimination or substitution step is valid, then find x^2-y^2 without expanding a new equation.

WORKED REASONING

Multiply the second equation by 6 to clear its constant denominators: $3x-2y=6$.

This is reversible because 6 is nonzero. Multiply the first equation by 2 and the transformed second by 3: $4x+6y=34$ and $9x-6y=18$. Add to obtain $13x=52$, so $x=4$. Substitute into $2x+3y=17$: $8+3y=17$, hence $y=3$. Check the second original equation: $4/2-3/3=1$. Use difference of squares: $x^2-y^2=(x-y)(x+y)=1\cdot7=7$.

WORKED SOLUTIONS

Week 1 · Intensive algebra

MISCONCEPTION ANALYSIS

Q15

[3 marks]

A student claims $3/(x-1)-2/(x+1)=1/(x^2-1)$, saying they “subtracted the numerators and multiplied the denominators”. Identify the invalid step, find the correct single fraction and state its domain.

WORKED REASONING

Numerators can be subtracted only after the fractions share a denominator. Use $(x-1)(x+1)=x^2-1$ and form $3(x+1)-2(x-1)=3x+3-2x+2=x+5$. The correct result is $(x+5)/(x^2-1)$ for $x\neq-1,1$. The student's $1/(x^2-1)$ loses the cross-multiplied terms.

WORKED SOLUTIONS

Week 1 · Intensive algebra

MISCONCEPTION ANALYSIS

Q16

[3 marks]

A student solves $(x^2-9)/(x-3)=6$ by multiplying through, gets $(x-3)^2=0$ and reports $x=3$. Is there a solution? Explain.

WORKED REASONING

The original denominator requires $x \neq 3$. For allowed x, the left side simplifies to $x+3$; $x+3=6$ gives $x=3$, which is excluded. Thus there is no solution. Multiplying by $x-3$ without keeping its nonzero condition can introduce this invalid candidate.

WORKED SOLUTIONS

Week 1 · Intensive algebra

MISCONCEPTION ANALYSIS

Q17

[3 marks]

A student writes $3^x+3^x=3^{2x}$ and therefore cannot solve $3^x+3^x=54$. Repair the step and solve the equation.

WORKED REASONING

Addition of equal terms doubles the term: $3^x+3^x=2\cdot3^x$, not a product of powers.

Thus $2\cdot3^x=54$, so $3^x=27=3^3$ and $x=3$. Check: $27+27=54$.

WORKED SOLUTIONS

Week 1 · Intensive algebra

STRETCH

Q18

[4 marks]

Suppose $a>0$ and $2^{x+1}=a\cdot 4^{x}$. Find a so that $x=-2$ is a solution. Show that for this a the solution is unique, using only index laws.

WORKED REASONING

At $x=-2$, $2^{-1}=a\cdot 4^{-2}$, so $1/2=a/16$ and $a=8$. Then $2^{x+1}=8\cdot 2^{2x}=2^{2x+3}$.

Equal powers of 2 give $x+1=2x+3$, hence $x=-2$. Since base 2 is injective, there is exactly one solution; no logarithms are needed.

WORKED SOLUTIONS

Week 1 · Intensive algebra

STRETCH

Q19

[5 marks]

For a real parameter k, solve $(x+1)/(x-2)-(x-1)/(x+2)=k/(x^2-4)$ for x. Determine the two values of k for which the equation has no solution, and explain why.

WORKED REASONING

Original restrictions: $x \neq 2,-2$. Multiply by $x^2-4=(x-2)(x+2)$, which is nonzero on the permitted domain. The left numerator is $(x+1)(x+2)-(x-1)(x-2)=(x^2+3x+2)-(x^2-3x+2)=6x$. Thus $6x=k$, giving candidate $x=k/6$. It is a solution only if $k/6 \neq 2,-2$, or $k \neq 12,-12$. For $k=12$ or -12, the sole candidate is an excluded denominator zero, so there is no solution.