G3 MATHEMATICS HOLIDAY REVISION

Week 2

Geometry under pressure

Worked solutions

Upper Secondary - Sec 3 and Sec 4

12 original questions

Attempt the student worksheet first. Then compare each step with the reasoning and checks in this companion.

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WORKED SOLUTIONS

Week 2 · Geometry under pressure

WARM-UPS

Q1

[3 marks]

A right triangle has perpendicular sides 9 cm and 12 cm. Find its hypotenuse and the sine of the acute angle opposite the 9 cm side.

WORKED REASONING

1. Pythagoras gives hypotenuse² = 9² + 12² = 225, so the hypotenuse is 15 cm.

2. For the specified angle, sin θ = opposite/hypotenuse = 9/15 = 3/5.

CHECK THIS STEP FIRST

Do not use 12 cm as the opposite side when the angle is stated to face the 9 cm side.

WORKED SOLUTIONS

Week 2 · Geometry under pressure

WARM-UPS

Q2

[3 marks]

Two similar triangles have corresponding side lengths in the ratio 3:5 (small:large). The smaller area is 27 cm². Find the larger area and the difference between their areas.

WORKED REASONING

1. The area ratio is $3^2:5^2 = 9:25$.

2. Larger area = $27 \times 25/9 = 75$ cm². The difference is $75 - 27 = 48$ cm².

CHECK THIS STEP FIRST

A length ratio is not an area ratio.

WORKED SOLUTIONS

Week 2 · Geometry under pressure

CORE APPLICATIONS

Q3

[4 marks]

In isosceles triangle ABC, AB = AC. The exterior angle formed by extending BC beyond C is 128°. Find all three interior angles and give a reason for each step.

WORKED REASONING

1. Interior angle C and its adjacent exterior angle lie on a straight line, so C = 180° − 128° = 52°.

2. AB = AC, so the base angles opposite them are equal: B = C = 52°.

3. Triangle angles sum to 180°, hence A = 180° − 52° − 52° = 76°.

CHECK THIS STEP FIRST

The given 128° is outside the triangle, not angle C.

WORKED SOLUTIONS

Week 2 · Geometry under pressure

CORE APPLICATIONS

Q4

[5 marks]

Two sides of a triangle are 7 cm and 9 cm with included angle 60°. Find the third side exactly and the triangle's area exactly. State why the sine rule is not the first step for the third side.

WORKED REASONING

1. No known side is paired with its opposite angle, so use the cosine rule.

2. Third side c satisfies $c^2 = 7^2 + 9^2 - 2(7)(9)\cos 60° = 67$. Thus $c = \sqrt{67}$ cm.

3. Area = $1/2 \times 7 \times 9 \times \sin 60° = 63\sqrt{3}/4$ cm².

CHECK THIS STEP FIRST

The 60° angle is included between 7 and 9; it is opposite neither of those sides.

WORKED SOLUTIONS

Week 2 · Geometry under pressure

CORE APPLICATIONS

Q5

[4 marks]

From a starting point, a hiker walks 6 km due north and then 8 km due east. Find the straight-line distance and bearing of the finish from the start. Give the bearing to 1 decimal place.

WORKED REASONING

1. The two legs are perpendicular, so displacement = $\sqrt{(6^2 + 8^2)} = 10$ km.

2. Measure clockwise from north: $\tan\theta$ = east/north = 8/6, so $\theta \approx 53.1°$.

3. The bearing is 053.1°. The reverse bearing, if needed, would be 233.1°.

CHECK THIS STEP FIRST

A bearing is measured from north, not from the eastward leg.

WORKED SOLUTIONS

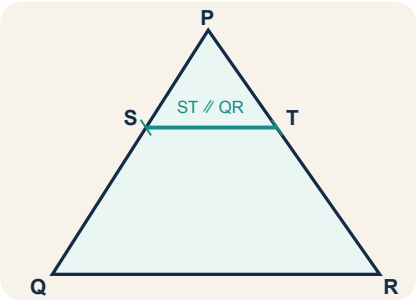
Week 2 · Geometry under pressure

CORE APPLICATIONS

Q6

[5 marks]

In triangle PQR, S is on PQ and T is on PR, with ST parallel to QR. Given PS:SQ = 2:3, area(PQR) = 125 cm² and QR = 15 cm, find ST and the area of trapezium SQRT.



WORKED REASONING

1. PS:PQ = 2:(2+3) = 2:5. Parallel lines give similar triangles PST and PQR.

2. ST/QR = 2/5, so ST = (2/5)(15) = 6 cm.

3. The small-to-large area ratio is (2/5)² = 4/25. Area(PST) = 125 × 4/25 = 20 cm².

4. Trapezium area = 125 − 20 = 105 cm².

CHECK THIS STEP FIRST

Use PS:PQ, not PS:SQ, as the scale factor for the two complete triangles.

WORKED SOLUTIONS

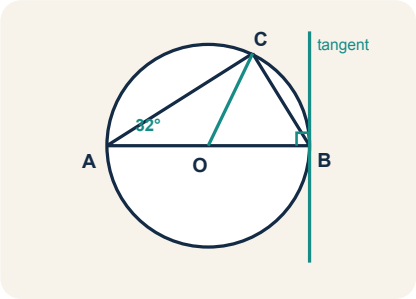
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CHALLENGING REASONING

Q7

[5 marks]

A and B are endpoints of a diameter of a circle with centre O, and C is another point on the circle. Angle BAC is 32°. Find angles ACB, ABC and the smaller central angle BOC. A tangent is drawn at B: find the acute angle between this tangent and BC using the radius–tangent right angle.



WORKED REASONING

1. Angle ACB is subtended by diameter AB, so ACB = 90° (angle in a semicircle).

2. Angle ABC = 180° − 90° − 32° = 58° by the triangle angle sum.

3. The smaller central angle BOC stands on arc BC, so BOC = 2(32°) = 64° (centre angle twice circumference angle on the same arc).

4. OB = OC, so base angle OBC = (180°−64°)/2 = 58°. The tangent at B is perpendicular to radius OB; hence its acute angle with BC is 90°−58° = 32°.

CHECK THIS STEP FIRST

Use the radius–tangent right angle and the named arc; no tangent–chord theorem is needed.

WORKED SOLUTIONS

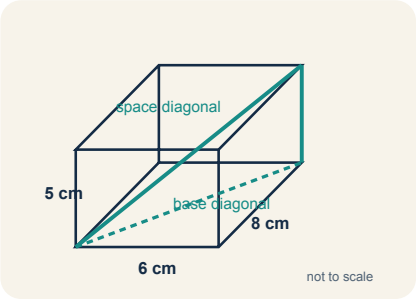
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CHALLENGING REASONING

Q8

[5 marks]

A rectangular box has length 6 cm, width 8 cm and vertical height 5 cm. Find the base diagonal, the space diagonal from one lower corner to the opposite upper corner, and the angle that the space diagonal makes with the horizontal base. Give the angle to 1 decimal place.



WORKED REASONING

1. In the base plane, diagonal = $\sqrt{(6^2 + 8^2)} = 10$ cm.

2. The space diagonal is the hypotenuse of a right triangle with horizontal side 10 and vertical side 5: $\sqrt{(10^2 + 5^2)} = 5\sqrt{5}$ cm.

3. For the angle with the base, $\tan\theta$ = vertical/horizontal = 5/10, so $\theta \approx 26.6°$.

CHECK THIS STEP FIRST

Use the base diagonal, not a single box edge, as the horizontal side of the final triangle.

WORKED SOLUTIONS

Week 2 · Geometry under pressure

CHALLENGING REASONING

Q9

[5 marks]

In triangle ABC, angle A = 30°, BC = 6 cm and AC = 10 cm. Find every possible value of angle B to 1 decimal place. Explain whether both values form a valid triangle.

WORKED REASONING

1. BC is opposite A and AC is opposite B. Sine rule gives sin B/10 = sin 30°/6, so sin B = 5/6.
2. The acute solution is B ≈ 56.4°. The supplementary solution is B ≈ 123.6°.
3. Their respective third angles are 180° − 30° − 56.4° = 93.6° and 180° − 30° − 123.6° = 26.4°. Both are positive, so both triangles are valid.

CHECK THIS STEP FIRST

Inverse sine returns one angle; check its supplement when the given information is side–side–angle.

WORKED SOLUTIONS

Week 2 · Geometry under pressure

MISCONCEPTION ANALYSIS

Q10

[4 marks]

In triangle ABC, AB = 5 cm and AC = 7 cm enclose angle BAC = 60°. A student writes 5/sin 60° = 7/sin B to find BC. Identify the first invalid step and find BC correctly.

WORKED REASONING

1. AB = 5 cm is opposite angle C, not angle A = 60°. The student has mismatched a side and angle; no known opposite pair is available yet.
2. Use the cosine rule for BC: $BC^2 = 5^2 + 7^2 - 2(5)(7)\cos 60° = 39$.
3. Hence BC = $\sqrt{39}$ cm (about 6.24 cm).

CHECK THIS STEP FIRST

Always mark the side opposite each angle before writing a sine-rule ratio.

WORKED SOLUTIONS

Week 2 · Geometry under pressure

STRETCH

Q11

[5 marks]

A circle has radius 6 cm. A minor sector has central angle 120°. Find its arc length, sector area and the area of the minor segment bounded by the chord, all exactly.

WORKED REASONING

1. Convert 120° to 2π/3 radians. Arc length = rθ = 6(2π/3) = 4π cm.

2. Sector area = (1/2)r²θ = (1/2)(36)(2π/3) = 12π cm².

3. The triangle formed by the two radii has area (1/2)(6)(6)sin 120° = 9√3 cm².

4. Minor segment area = sector − triangle = 12π − 9√3 cm².

CHECK THIS STEP FIRST

Subtract the triangle from the sector for the minor segment; do not subtract the entire circle.

WORKED SOLUTIONS

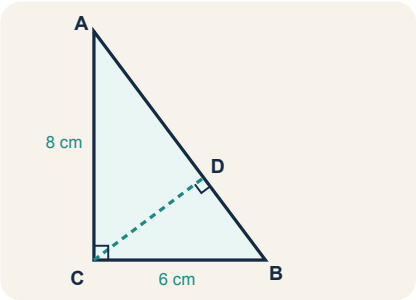
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STRETCH

Q12

[6 marks]

Triangle ABC is right-angled at C, with AC = 8 cm and BC = 6 cm. D lies on hypotenuse AB and CD is perpendicular to AB. Find AB, AD, DB and CD. Show that AD×DB = CD² and explain which similar triangles justify the result.



WORKED REASONING

1. Pythagoras gives AB = √(8²+6²) = 10 cm.

2. Triangles ACD, CBD and ABC are similar by their matching acute angles and right angles. In ACD and ABC, AD/AC = AC/AB, so AD = 8²/10 = 6.4 cm. In CBD and ABC, DB/BC = BC/AB, so DB = 6²/10 = 3.6 cm.

3. The area of ABC is (1/2)(8)(6) = 24 cm², also (1/2)(10)(CD). Hence CD = 4.8 cm.

4. Triangles ACD and CBD also give AD/CD = CD/DB, so AD×DB = CD². Check numerically: 6.4×3.6 = 23.04 = 4.8², and AD+DB = 10 cm.

CHECK THIS STEP FIRST

The perpendicular altitude CD is not the midpoint of AB; derive segment lengths from similarity, then check they sum to AB.