G3 MATHEMATICS HOLIDAY REVISION

Week 4

Exam mode

Worked solutions

Upper Secondary - Sec 3 and Sec 4

15 original questions

Attempt the student worksheet first. Then compare each step with the reasoning and checks in this companion.

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WORKED SOLUTIONS

Week 4 · Exam mode

TIMED CHECKS

Q1

[4 marks]

Solve the compound inequality $3x - 5 < 2x + 4 \leq 4x + 10$. Write the answer as one interval and test one boundary.

WORKED REASONING

1. From $3x-5 < 2x+4$, subtract $2x$ and add 5 to get $x < 9$.
2. From $2x+4 \leq 4x+10$, subtract $2x$ and 10 to get $-6 \leq 2x$, so $x \geq -3$.
3. Together, $-3 \leq x < 9$. At $x = -3$, both original comparisons are true; $x = 9$ fails the strict first comparison.

CHECK THIS STEP FIRST

Solve each comparison and take their intersection, not their union.

WORKED SOLUTIONS

Week 4 · Exam mode

TIMED CHECKS

Q2

[4 marks]

Solve $4^{x+1} = 8^{2x-1}$. Give an exact answer and show a common-base check.

WORKED REASONING

1. Rewrite 4 as 2^2 and 8 as 2^3: $2^{2x+2} = 2^{6x-3}$.

2. Equal positive-base powers have equal exponents, so $2x+2 = 6x-3$ and $4x = 5$.

3. Thus $x = 5/4$. The exponents are both $9/2$ at this value, so the powers agree.

CHECK THIS STEP FIRST

The outside exponents multiply the original exponents 2 and 3; do not add them.

WORKED SOLUTIONS

Week 4 · Exam mode

MIXED APPLICATIONS

Q3

[5 marks]

A tank starts empty. Water enters at 6 litres per minute for 8 minutes, then at 4 litres per minute for 12 minutes. The tank holds 120 litres. How much more water is needed, and how long will that take at 3 litres per minute?

WORKED REASONING

1. The first stage adds $6\times8 = 48$ litres. The second adds $4\times12 = 48$ litres.
2. The tank contains 96 litres, so it needs $120-96 = 24$ litres more.
3. At 3 litres per minute, the remaining time is $24/3 = 8$ minutes. Check that $48+48+24=120$ litres.

CHECK THIS STEP FIRST

A flow rate multiplies time to give volume; do not add rates from stages that occur one after another.

WORKED SOLUTIONS

Week 4 · Exam mode

MIXED APPLICATIONS

Q4

[4 marks]

A bag has 3 red and 2 blue counters. Two are drawn without replacement. Find the probability of exactly one blue counter, showing the different possible orders.

WORKED REASONING

1. Red then blue has probability (3/5)(2/4) = 3/10.

2. Blue then red has probability (2/5)(3/4) = 3/10.

3. The orders are disjoint, so add: 3/10+3/10 = 3/5.

CHECK THIS STEP FIRST

Multiply within one order; add the probabilities of the two valid orders.

WORKED SOLUTIONS

Week 4 · Exam mode

MIXED APPLICATIONS

Q5

[6 marks]

A(−1,2) and B(3,10) are endpoints of a segment. Find its midpoint, the equation of its perpendicular bisector and where that bisector meets the x-axis.

WORKED REASONING

1. Midpoint M = ((−1+3)/2,(2+10)/2) = (1,6).

2. Gradient AB = (10−2)/(3−(−1)) = 2, so a perpendicular line has gradient −1/2.

3. Through M: y−6 = −(1/2)(x−1), or y = −x/2+13/2.

4. On the x-axis y=0, so x=13. The intersection is (13,0).

CHECK THIS STEP FIRST

The bisector passes through the midpoint, not necessarily through either endpoint.

WORKED SOLUTIONS

Week 4 · Exam mode

MIXED APPLICATIONS

Q6

[4 marks]

A map has scale 1:25 000. A path measures 7.2 cm on the map. A square park has side 2 cm on the same map. Find the path length in kilometres and the actual park area in hectares. (1 hectare = 10 000 m².)

WORKED REASONING

1. Actual path length = 7.2×25 000 cm = 180 000 cm = 1.8 km.

2. Actual park side = 2×25 000 cm = 50 000 cm = 500 m.

3. Actual area = 500² = 250 000 m² = 25 hectares.

CHECK THIS STEP FIRST

Convert the map side length before squaring; area does not use the linear scale factor once.

WORKED SOLUTIONS

Week 4 · Exam mode

MIXED APPLICATIONS

Q7

[5 marks]

A discrete data set has values 1, 2 and 3 with respective frequencies 2, 5 and 3. Find the mean and standard deviation, using population standard deviation for these ten values.

WORKED REASONING

1. There are 2+5+3 = 10 values. Sum of values = 2(1)+5(2)+3(3)=21, so mean = 21/10 = 2.1.

2. Sum of squares = $2(1^2)+5(2^2)+3(3^2)=49$, so mean square = 49/10 = 4.9.

3. Variance = $4.9-2.1^2 = 0.49$. Standard deviation = $\sqrt{0.49} = 0.7$.

CHECK THIS STEP FIRST

Use frequencies in both sums; do not divide by the three distinct values.

WORKED SOLUTIONS

Week 4 · Exam mode

MIXED APPLICATIONS

Q8

[5 marks]

A shop sells pens at $2 each and notebooks at $5 each. Its sales matrix has rows for Day 1 and Day 2 and columns for pens and notebooks: [[3,2],[1,4]]. Choose a price vector, use matrix multiplication to find each day's revenue, and find the two-day total.

WORKED REASONING

1. Use the 2×1 price column [[2],[5]] dollars. The 2×2 sales matrix times this column gives a 2×1 revenue column.
2. Day 1 revenue = 3(2)+2(5) = $16. Day 2 revenue = 1(2)+4(5) = $22.
3. Total revenue = 16+22 = $38. A price row multiplied on the wrong side would not represent the stated day-by-item layout.

CHECK THIS STEP FIRST

Match matrix columns to the order of entries in the price column before multiplying.

WORKED SOLUTIONS

Week 4 · Exam mode

MIXED APPLICATIONS

Q9

[5 marks]

A cylindrical container of radius 3 cm and height 10 cm is open at the top. It is filled to 80% of capacity. Find the volume of water and the outside area to be painted if only the curved surface and base are painted. Give exact answers.

WORKED REASONING

1. Full capacity is $\pi r^2 h = \pi(3^2)(10) = 90\pi$ cm³. Water volume = $0.80(90\pi) = 72\pi$ cm³.
2. Curved outside area = $2\pi rh = 2\pi(3)(10) = 60\pi$ cm².
3. Base area = $\pi r^2 = 9\pi$ cm². Painted area = $60\pi+9\pi = 69\pi$ cm²; the open top has no circular face to paint.

CHECK THIS STEP FIRST

Volume uses cubic units; painted area uses square units. Do not count the open top.

WORKED SOLUTIONS

Week 4 · Exam mode

MISTAKE ANALYSIS

Q10

[5 marks]

For $y = x^2 - 6x + 5$, find the turning point and x-intercepts. A student says the line $y = -3$ meets the curve only once because the turning point lies below it. Explain the error and find every intersection with $y = -3$.

WORKED REASONING

1. Complete the square: $y = (x-3)^2 - 4$, so the minimum turning point is $(3,-4)$.
2. Factor: $y = (x-1)(x-5)$, giving x-intercepts $(1,0)$ and $(5,0)$.
3. Since -3 is above the minimum -4, a horizontal line at $y = -3$ can meet both sides of the parabola. Set $x^2 - 6x + 5 = -3$, so $(x-2)(x-4)=0$.
4. The intersections are $(2,-3)$ and $(4,-3)$. The student claim ignores the two symmetric branches.

CHECK THIS STEP FIRST

A line through the turning point would touch once; a higher horizontal line crosses twice for this upward-opening curve.

WORKED SOLUTIONS

Week 4 · Exam mode

MISTAKE ANALYSIS

Q11

[4 marks]

A student writes $(x^2-4)/(x-2)=x+2$ and then says $x=2$ is a solution of $(x^2-4)/(x-2)=x+2$ because both sides equal 4. Identify the error and state the complete set of real solutions to the original equation.

WORKED REASONING

1. The original denominator is zero at $x=2$, so $x=2$ is not in the domain.
2. For $x\neq2$, factor $x^2-4=(x-2)(x+2)$ and cancel the nonzero factor $x-2$. Both sides then equal $x+2$.
3. The equation holds for every real x except $x=2$. Substitution into a simplified form cannot restore an excluded value.

CHECK THIS STEP FIRST

Record the original restriction before cancellation.

WORKED SOLUTIONS

Week 4 · Exam mode

EXTENDED EXAM TASKS

Q12

[8 marks]

A rectangular community garden has perimeter 40 m and area 96 m². A 2 m-wide entrance is left unfenced. Fencing costs $12 per metre before a 10% discount on the fencing bill. Find the garden's dimensions, its diagonal exactly, and the final fencing cost. Check that the dimensions fit both original conditions.

WORKED REASONING

1. Let the shorter side be x metres. Perimeter 40 m gives longer side $20-x$ metres, with $0<x\leq 10$.
2. Area: $x(20-x)=96$, so $x^2-20x+96=0$ and $(x-8)(x-12)=0$. Since x is the shorter side, $x=8$ and the longer side is 12 m.
3. Pythagoras gives the diagonal $\sqrt{(8^2+12^2)}=\sqrt{208}=4\sqrt{13}$ m.
4. The fence length is the perimeter minus the entrance: $40-2=38$ m. Before discount it costs $38\times\$12=\$456$.
5. After the 10% discount, the bill is $0.90\times\$456=\$410.40$. Check: $2(8+12)=40$ m and $8\times12=96$ m².

CHECK THIS STEP FIRST

Apply the entrance deduction to the perimeter before costing, then apply the discount to the bill; do not discount the side lengths.

WORKED SOLUTIONS

Week 4 · Exam mode

EXTENDED EXAM TASKS

Q13

[8 marks]

Printer A completes a booklet order on its own in x hours. Printer B takes 3 hours longer than printer A to complete the same order. Together they complete one order in 2 hours. Find x, showing why any rejected algebraic root is impossible. On a second identical order, both printers run for 1 hour before A stops. How much longer does B need to finish it? Check your result using fractions of an order completed.

WORKED REASONING

1. The hourly rates are 1/x and 1/(x+3) orders, with x>0. Together: 1/x + 1/(x+3) = 1/2.
2. Multiply by 2x(x+3): 2(x+3)+2x=x(x+3), hence $x^2-x-6=0$.
3. Factor (x−3)(x+2)=0. The roots are 3 and −2; reject −2 because a completion time must be positive. Printer A takes 3 hours and B takes 6 hours.
4. Together they complete 1/2 of the second order in the first hour, leaving 1/2. B alone completes 1/6 of an order per hour, so the extra time is (1/2)/(1/6)=3 hours.
5. Check: A completes 1/3 in the first hour; B completes 1/6 then and 3(1/6)=1/2 later. Total 1/3+1/6+1/2=1 order.

CHECK THIS STEP FIRST

Use rates in orders per hour, not completion times, and reject a negative root before interpreting the remaining stage.

WORKED SOLUTIONS

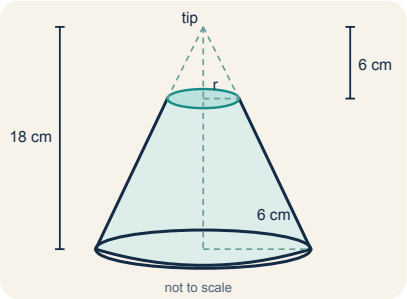
Week 4 · Exam mode

EXTENDED EXAM TASKS

Q14

[9 marks]

A solid right circular cone has height 18 cm and base radius 6 cm. A plane parallel to the base cuts off a smaller cone of height 6 cm at the tip. Find the radius of the new circular top face and the exact volume of the remaining solid. Find its total surface area, including both circular faces. A student multiplies the original cone's volume by 12/18 because the remaining solid is 12 cm high. Explain why this is invalid.



WORKED REASONING

1. The removed tip cone is similar to the original cone. Its height scale is $6/18=1/3$, so its radius is $(1/3)(6)=2$ cm.
2. Full cone volume = $(1/3)\pi(6^2)(18)=216\pi$ cm³. Removed cone volume = $(1/3)\pi(2^2)(6)=8\pi$ cm³. Remaining volume = $216\pi-8\pi=208\pi$ cm³.
3. Use Pythagoras for slant heights: original $l=\sqrt{(18^2+6^2)}=6\sqrt{10}$ cm; removed tip $l=\sqrt{(6^2+2^2)}=2\sqrt{10}$ cm.
4. Remaining curved area = $\pi(6)(6\sqrt{10})-\pi(2)(2\sqrt{10})=32\pi\sqrt{10}$ cm². The bottom face is 36π cm² and the new top face is 4π cm².
5. Total surface area = $(32\sqrt{10}+40)\pi$ cm². A 12/18 height fraction is not a volume fraction: the cut-off cone's volume scales by $(1/3)^3=1/27$, and the remaining volume is 26/27 of the original.

CHECK THIS STEP FIRST

Subtract the removed cone before calculating the remaining volume and curved area; include the new circular cut face in total surface area.

WORKED SOLUTIONS

Week 4 · Exam mode

EXTENDED EXAM TASKS

Q15

[9 marks]

The journey times of two groups of 40 students are grouped into 0–<10, 10–<20, 20–<30, 30–<40 and 40–<50 minutes. Group A has respective frequencies 4, p, 14, q, 2 and an estimated mean of 25 minutes. Group B has frequencies 8, 10, 10, 8, 4. Find p and q, estimate Group B's mean, and identify the median class of each group. A student claims Group A must have a higher exact median because its estimated mean is higher. Assess that claim. Finally, find the probability that a randomly chosen Group A student is in a journey-time class of at least 30 minutes.

WORKED REASONING

1. Group A totals 40, so p+q=20. The class midpoints are 5, 15, 25, 35 and 45 minutes.
2. Its estimated total is 40(25)=1000. Known midpoint contributions are 4(5)+14(25)+2(45)=460, so 15p+35q=540.
3. Substitute p=20−q: 15(20−q)+35q=540, giving 20q=240. Thus q=12 and p=8; the frequencies 4+8+14+12+2 total 40.
4. Group B's midpoint total is 8(5)+10(15)+10(25)+8(35)+4(45)=900, so its estimated mean is 900/40=22.5 minutes.
5. For 40 values the middle positions are 20 and 21. Group A cumulative frequencies are 4,12,26,38,40; Group B's are 8,18,28,36,40. Both median classes are 20–<30 minutes.
6. The claim is not justified: grouped classes do not reveal the exact median values, and an estimated mean comparison does not determine their ordering. In Group A, 12+2=14 of 40 journeys are in classes starting at 30 minutes, so the probability is 14/40=7/20.

CHECK THIS STEP FIRST

A higher estimated mean does not establish a higher exact median; grouped data only locate both medians within their classes.